Best Performance of Reactor Controllers Using Stochastic Optimization Method

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Abstract

Based on the mass and energy balances for the reactor and heating system, a mathematical model for a continuous stirred tank reactor is created. The concentration is changed stepwise, and the reactor's temperature is gauged as a result. This study compares the use of PI, generic model control, and fuzzy logic controllers on the system with the aim of evaluating each one's performance in light of the integral of the absolute error that is produced. The controller's settings are adjusted using a simulated annealing technique. However, in order to have a fairly comparison The range of the PI and Generic model controller's gains are increased as well as the simulated annealing solution numbers, on the other hand the number of membership functions for variable and solution numbers are increase for fuzzy controller. MATLAB/SIMULINK has been used to implement the control and simulation investigation.

Keywords: *Mathematical Mödling of continuous stirred tank reactor, MATLAB Simulation, PID controller, Generic Model Control, Fuzzy Logic Control, and Simulated Annealing (SA).*

1. Introduction

The most significant component used for unit operations in a chemical plant is a continuous stirred tank reactor system (CSTR). Basically, the nonlinear dynamic characteristic of a chemical reactor system is the complexity. Its state estimation and real-time control based on mathematical modeling have attracted a lot of interest. However, it has proven challenging to establish an effective control approach due to a lack of understanding of the dynamics of the process and the reactor's highly sensitive and nonlinear behavior. Only with an accurate model can the CSTR be controlled effectively [1]. Reaction phase (homogenous reactor, heterogeneous reactor), or operating modes (continuous stirred tank reactor, batch reactor, semi-batch, and so on) are two ways to categorize chemical reactors.

2. Mathematical Model of the continuous Stirred Tank reactor

A mathematical model of a continuous stirred tank reactor is developed depending on mass and energy balances. A summing a first order irreversible exothermic reaction ($A \rightarrow B$) is taken place in a continuous Stirred Tank Reactor as shown in Figure (1), the heat generated by the reaction is removed using a cooling coil inside the reactor. Perfectly mixing is assumed in CSTR and the change in volume due to reaction is negligible. The reactor mass and energy equations are:

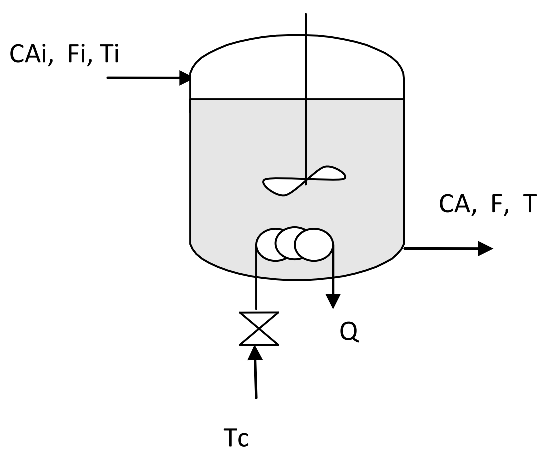


Figure 1. Continuous Stirred Tank Reactor

Over all mass balance

$$\frac{dV}{dt} = F_i - F \tag{1}$$

and $$F_i = F \tag{2}$$

F_i, F are inlet, outlet flow, V reactor volume, t is the time, C_{Ai}, C_A inlet, outlet concentration of component A, T_i, T inlet, outlet temperature, r is reaction rate, E is activation energy, R is gas constant, k_0 is pre-exponential constant, ρ is density, C_p, specific heat capacity, H_r heat of reaction, T_c coolant temperature, and UA is a product of heat transfer coefficient and area.

Component (A) mass balance

$$\frac{dVC_A}{dt} = F_i C_{Ai} - FC_A - rV \tag{3}$$

Where r is the rate of a first order reaction

$$r = k_0 e^{\frac{-E}{RT}} C_A \tag{4}$$

and V is constant then (3) can be written as:

$$\frac{dC_A}{dt} = \frac{F}{V} C_{Ai} - \frac{F}{V} C_A - k_0 e^{\frac{-E}{RT}} C_A \tag{5}$$

Heat balance

$$\frac{\rho dVC_p T}{dt} = \rho C_p F_i T_i - \rho C_p FT - H_r VC_A k_0 e^{\frac{-E}{RT}} - UA(T - T_c) \tag{6}$$

Where, V is constant, and the specific heat C_p is a function of temperature then from (2), and (6).

$$\frac{dT}{dt} = \frac{F}{V}(T_i - T) - \frac{H_r C_A k_0 e^{\frac{-E}{RT}}}{\rho C_p} - \frac{UA}{\rho C_p V}(T - T_c) \tag{7}$$

3. Controllers

3.1. PID Controller

PID controller is the most basic type of controller that uses the system's Derivative and Integral operations. PID controllers perform a number of crucial tasks, such as eliminating steady-state error by integral action and managing actuator saturation when combined with anti-windup. These controllers are also efficient for a variety of control issues, especially when the process dynamics are benign and the performance demands are low [2]. The following equation is a representation of a PID controller.

$$C(t) = K_c\left(e(t) + \frac{1}{\tau_i}\int_0^t e(t)\,dt + \tau_D\frac{de(t)}{dt}\right) \tag{8}$$

Where:

K_c = Proportion al constant

τ_i = Time integral constant

τ_D = Derivative time constant

$e(t)$ = Error

$C(t)$ = Controller output

3.2. Generic Model Controller (GMC)

In recent years, it has become clear that greater process control is required. Generic model control (GMC), which has been shown to exhibit certain tolerance for a wide range of process nonlinearity against model mismatches [3], has attracted increasing interest since its first in 1987. Two tuning parameters can be used to get the appropriate response. Below are a few benefits that make GMC a good framework for creating system controllers.

The process model appears directly in the control algorithm.

a) GMC can cope with non-linearity inherited to processes.

b) GMC provides feedback control of the rate of change of the control variable.

c) The GMC algorithm is relatively easy to implement.

More details of GMC method can be found in [4]. Consider a process described by the following equation:

$$\dot{x} = f(x, u, d, t) \tag{9}$$

$$y = g(x) \tag{10}$$

Where x is the state variable, u is the manipulated variable, d is the disturbance variable t is the time, and y is the output. In general, f and g are some nonlinear functions. It follows from (9) and (10) that:

$$\dot{y} = G_x f(x, u, d, t) \tag{11}$$

For a specific desired steady state value, the GMC algorithm specifies a rate of change of the output variables as:

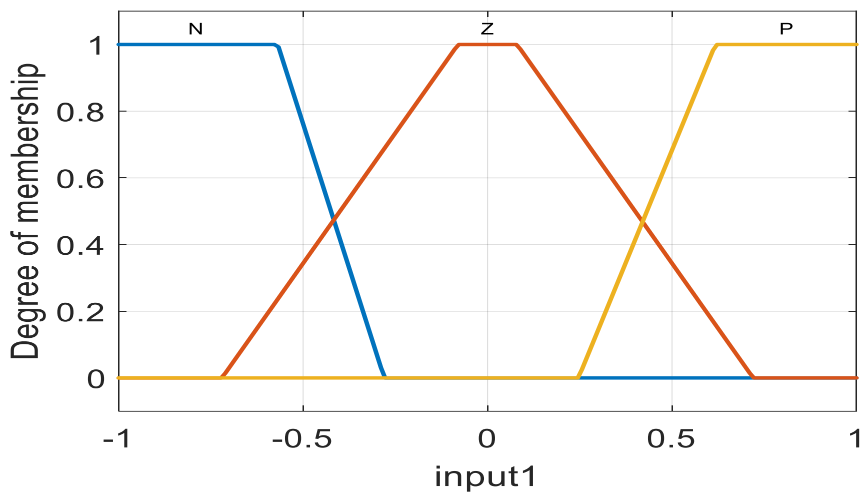
$$\dot{y} = K_1(y_{sp} - y) - K_2 \int (y_{sp} - y)\,dt \tag{12}$$

In (12), two process desires are obvious. First, when the system is at a greater distance from the setpoint, then the system should travel towards the set point more quickly. Moreover, the longer that the system has remained offset from the set point, then the system should also travel towards the set point more quickly. The values of K1 and K2 are what determine the speeds. Therefore, to solve for the control, the actual output rate is set equal to the desired output rate, in other words setting (11) equal to (12), giving the following equation from which the control, u, can be solved.

$$G_x f(x, u, d, t) = K_1(y_{sp} - y) - K_2 \int (y_{sp} - y)\,dt \tag{13}$$

3.3. Fuzzy Logic Controller

One of the most active and fruitful areas is fuzzy logic control [5]. Fuzzy control can be used effectively in poorly specified processes, as shown by FLC applications [6]. A set of linguistic control rules connected by the dual ideas of fuzzy implication and compositional rules of inference form the foundation of fuzzy logic, which is built on a spirit that is similar to human thinking [7]. FLC is different from traditional control approaches in that it uses a straightforward rule-based approach to address the control problem rather than mathematically modelling the system. It likewise makes use of erroneous data, but it is descriptive of what must occur [8]. Typical MFs of the controller are shown in Figures (2a) and (2b), thus, the quantity of MFs used for variable is 3, and then the number of rules required to map the input into the output is 3.



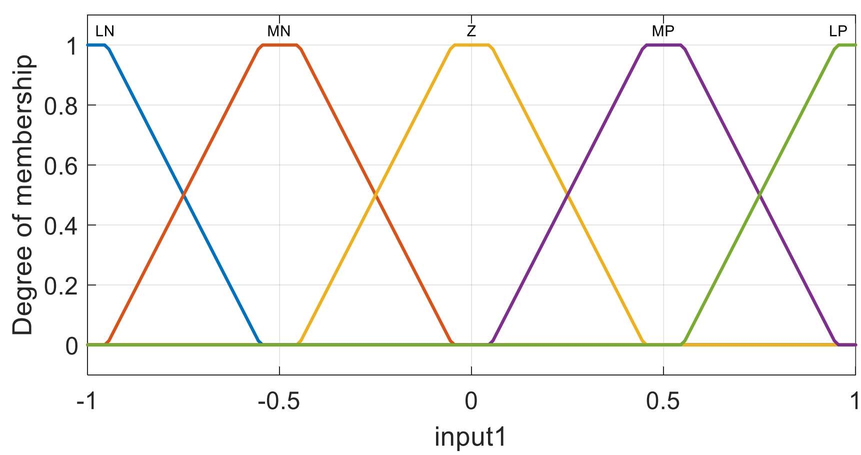


Figure 2b. Model of 5 membership

4. Simulated Annealing and Its Application to Controller Tuning

4.1. Solution representation

A global search technique called "simulated annealing" is based on an analogy with the solids' physical annealing process [9, 10, 11]. This optimization method has been utilized on a CSTR in the MATLAB and SIMULINK environments to fine-tune proportional integral (PI), generic model (GMC), and fuzzy controllers that are used to manage the temperature and concentration of the process. A randomly generated possible solution to a problem, Y, is compared to an existing solution, X, in the simulated annealing approach. The likelihood that Y will be approved for study depends on how close Y is to X and how developed the solution is, as indicated by a "temperature" parameter, Ts. According to a probability function that once again depends on the temperature Ts, both potential solutions are examined and Y is ultimately picked to replace X as the current answer. This idea is applied to the aforementioned controllers on the assumption that X and Y are potential solutions:

For PI parameters:

$$X = \left[K_{Px}, K_{Ix}\right] \qquad Y = \left[K_{Py}, K_{Iy}\right] \tag{14}$$

For GMC parameters:

$$X = \left[K_{1x}, K_{2x}\right] \qquad Y = \left[K_{1y}, K_{2y}\right] \tag{15}$$

For fuzzy controller:

$$X = \left[a_{ij} \ldots\ldots\ldots\ldots a_{nm}, b_{ij} \ldots\ldots\ldots\ldots\ldots\ldots b_{nm}\right]$$

$$Y = \left[a_{ij} \ldots\ldots\ldots\ldots a_{nm}, b_{ij} \ldots\ldots\ldots\ldots\ldots\ldots b_{nm}\right]$$

Where (16)

$i = 1 : n$

$j=1:m$

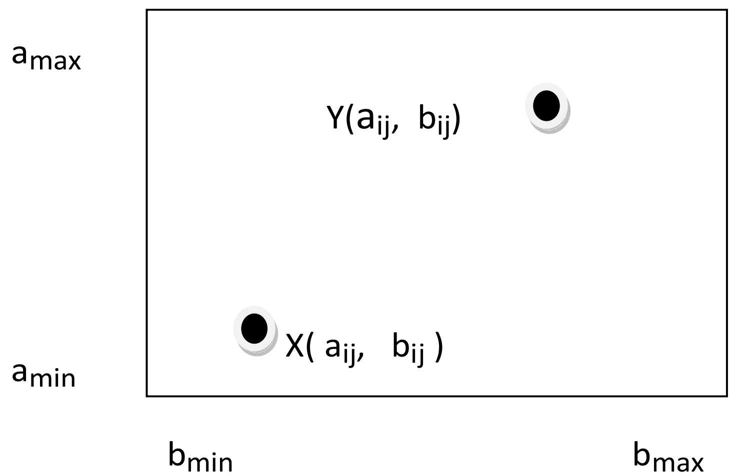


Figure 3. Search space

Where this solution must lie within the allowed system search space. The solution space defines the maximum and minimum values for the controller parameters or the universe of discourse for input and output membership functions for fuzzy controller which are represented by (a, and b) respectively, as shown in Figure 3.

4.2. Acceptance and swap probabilities

The range of the search is defined as:

$$\text{range} = \frac{\sqrt{(b_{max} - b_{min})^2 + (a_{max} - a_{min})^2}}{2} \tag{17}$$

The displacement is the distance between X and Y in the search space:

$$\text{displacement} = \sqrt{(Y - X)^2} \tag{18}$$

An acceptance probability, P_A, is then calculated:

$$P_A = \exp\left[-\left(\frac{\text{displaceme nt}}{\text{range}}\right) * \left(\frac{T_{st}}{Ts}\right)\right] \tag{19}$$

Where Ts is the instantaneous temperature, which is reduced as the solution proceeds and Tst is the initial temperature. P_A is compared to a random value r1 in the range [0 – 1]. If $r_1 > P_A$, the potential solution Y is rejected and the process is repeated. If $r_1 < P_A$, then Y is accepted for evaluation. The integral absolute error (IAE_Y) obtained from the process when using the controller gain values defined by Y is compared to the integral absolute error (IAE_X) obtained using the controller gain values defined by X in the swap probability function, P_S which is compare to a random values r_2

$$P_S = \frac{1}{1 + \exp\left\{\left(\frac{IAE_Y - IAE_X}{IAE_X}\right) * \left(\frac{T_{st}}{Ts}\right)\right\}} \tag{20}$$

in the range of [0 – 1]. If $r_2 > P_S$, then the original solution X is retained, and, if r_2 <, P_S, the new solution Y, is accepted, and X is replaced by Y. The concept of simulated annealing can be concluded from the following two figures. (4) and (5). Many alternatives for the decrement function of the temperature, Ts, are available. In this work, the decrement function proposed inversely proportional of the temperature to the number of potential solutions investigated.

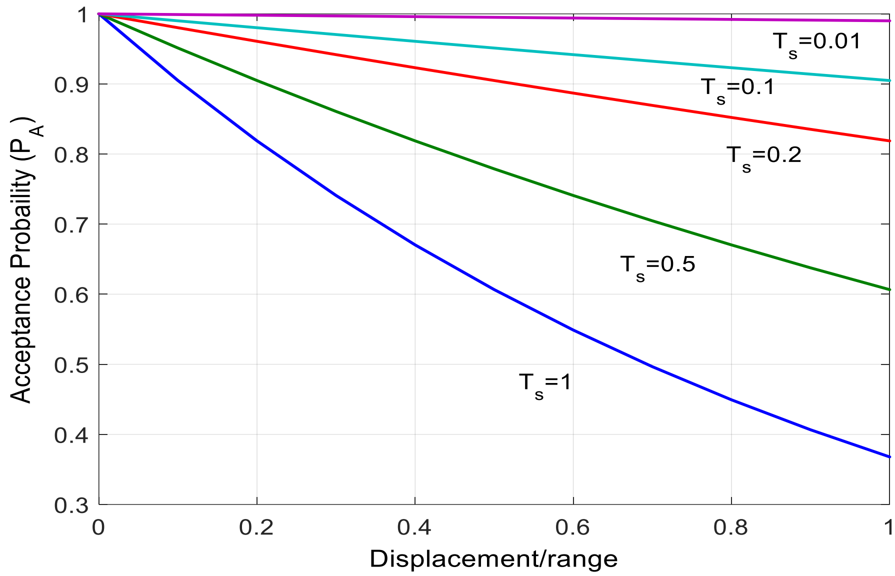


Figure 4. The relationship of acceptance probability (P_A), and the (displacement/range) at different values of (Ts)

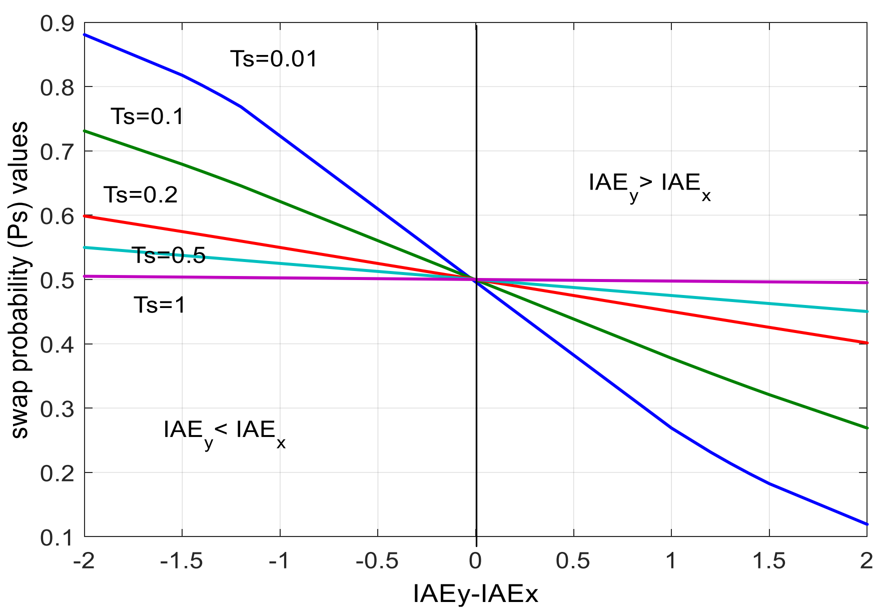


Figure 5. The relationship of swap probability Ps, and the values of IAE_Y, and IAE_X at different values of Ts.

5. Simulation

A technological computing environment for high performance numerical calculation is called MATLAB (matrix laboratory). A MATLAB extension is SIMULINK (Simulation and Link). It collaborates with MATLAB to provide graphical user interface (GUI)-based modelling, simulation, and analysis of dynamical systems. Figure 6 illustrates how the feedback control system can be expressed in a SIMULINK.

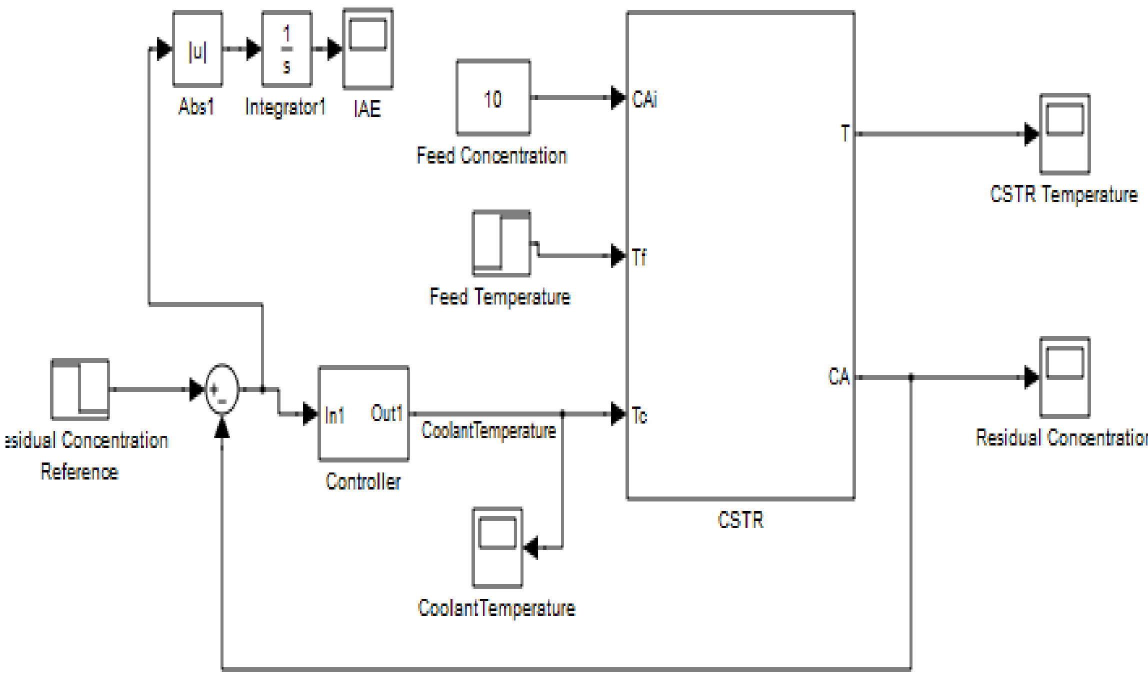


Figure 6. Feedback control system

A simulation study using a quick step change of decreasing the residual Concentration Reference at a time of (2 sec) by a magnitude of 0.5 examines the impact of the controllers on the CSTR temperature and a residual concentration. This is done in two scenarios, one without a feed temperature disturbance and the other with one applied at (t = 30 sec), resulting in a 5-degree abrupt spike in the feed temperature. A search range space of [-2000 2000] is used for both PI and

GMC gains, whereas, 3 membership function are used for variable in a fuzzy controller. However, when applying Simulated annealing algorithm, 1000 number of solutions where used for PI and GMC, but 3500 number of solutions for fuzzy controller because there are 8 points to be tuned not only two as in PI and GMC. However, In order to make a fairly comparison study between the three mentioned controllers, first, the search range of the PI and GMC controllers were, extended from [-2000 2000] to [-3000 3000] and keeping the number of solutions to 1000. While, for fuzzy controller to have a good and compatible performance extending the number of solutions were not enough. Therefore, the number of memberships of a variable has to be a signed to 5 MFs for each variable and the number of solutions to 5500.

5.1 Results

Below is an illustration of how the three different types of controllers operate. Figures (7, 8, 9) display the outcomes produced by standard trial and error settings. However, when using the stochastic simulated annealing optimization approach with a PI and GMC gain range of [-2000 2000] a number of solutions of 1000, and 3500 for PI, GMC, and Fuzzy controllers, the optimal IAE values respectively, are [0.1791 0.1693 0.2048]. For PI, GMC, and fuzzy controllers, the optimum solutions were discovered at simulation times [905 655 548], respectively.

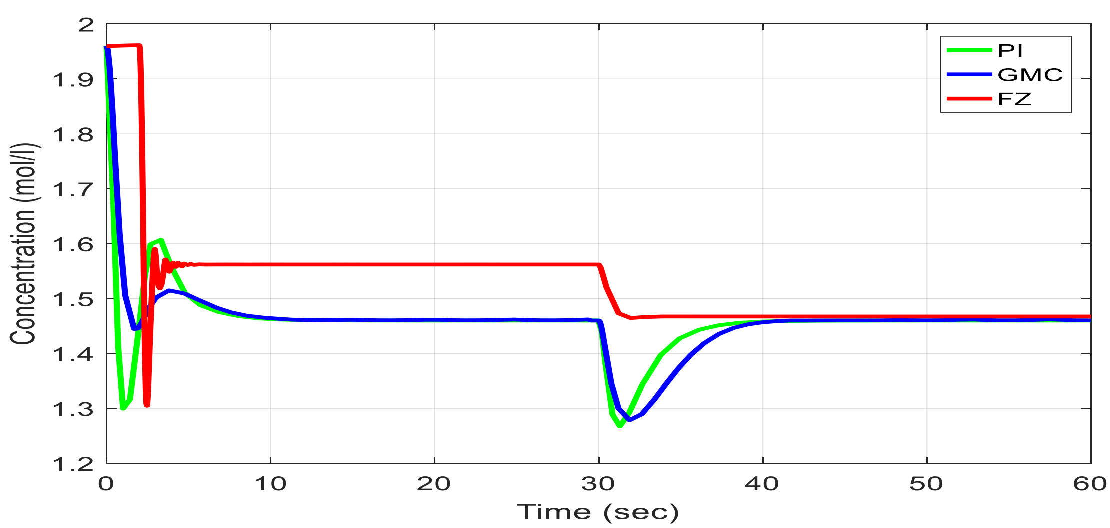


Figure 7. Concentration response of different controllers by conventional setting

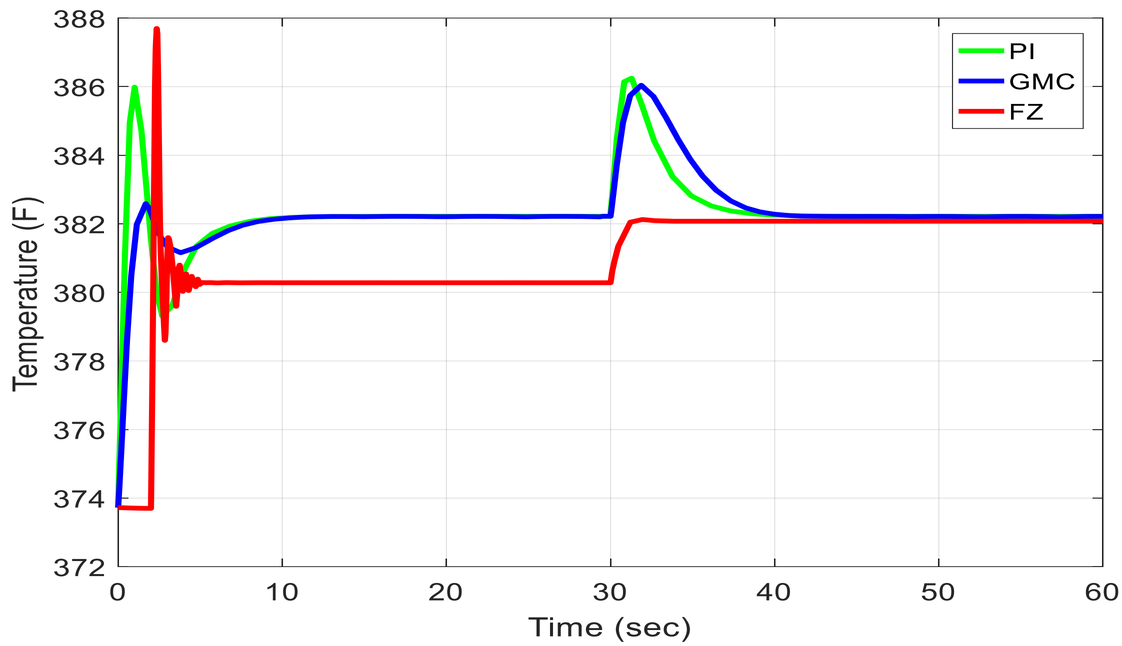


Figure 8. Temperature response of different Controllers by conventional settings

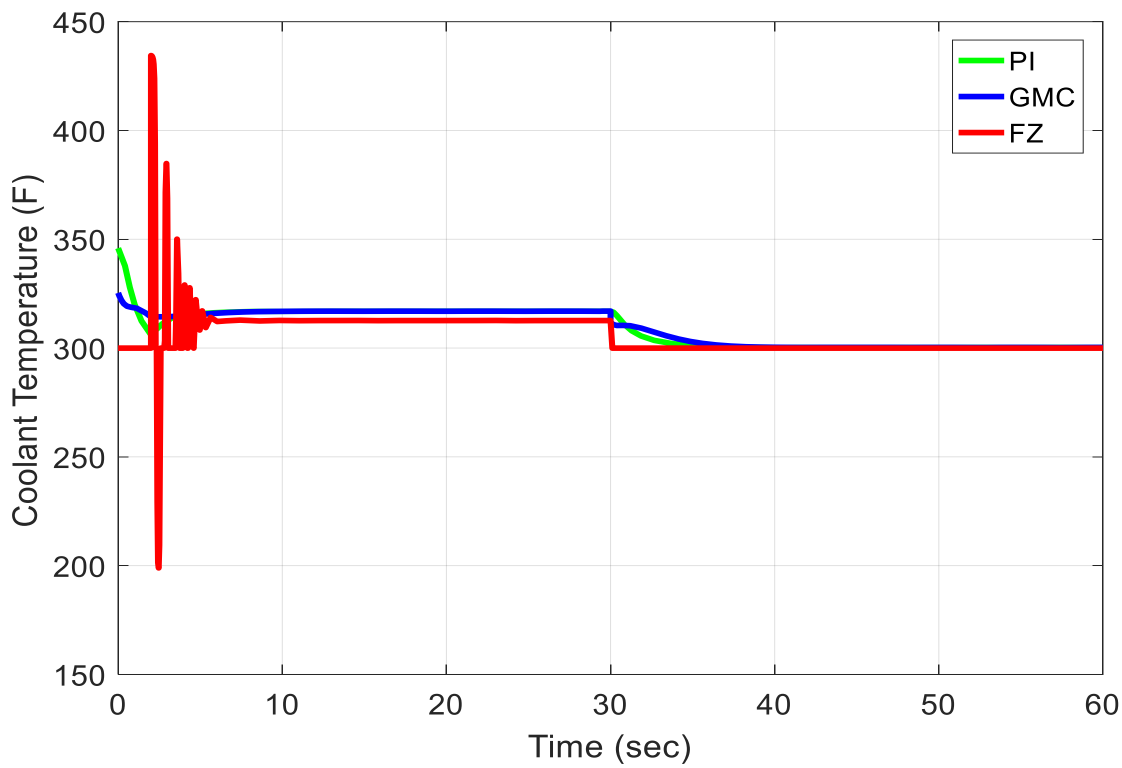


Figure 9. Coolant Temperature response for different Controllers by conventional settings

Figures [10, 11, 12, 13, 14, 15] and (16) show the outcomes obtained when using (SA) algorithm.

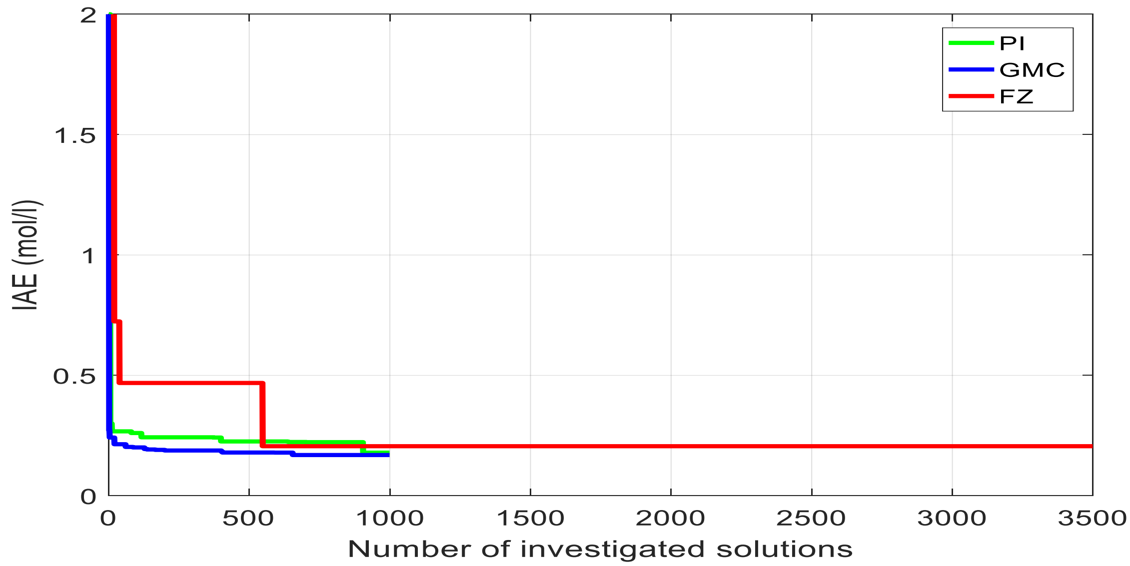


Figure 10. IAE obtained by SA for using different controllers

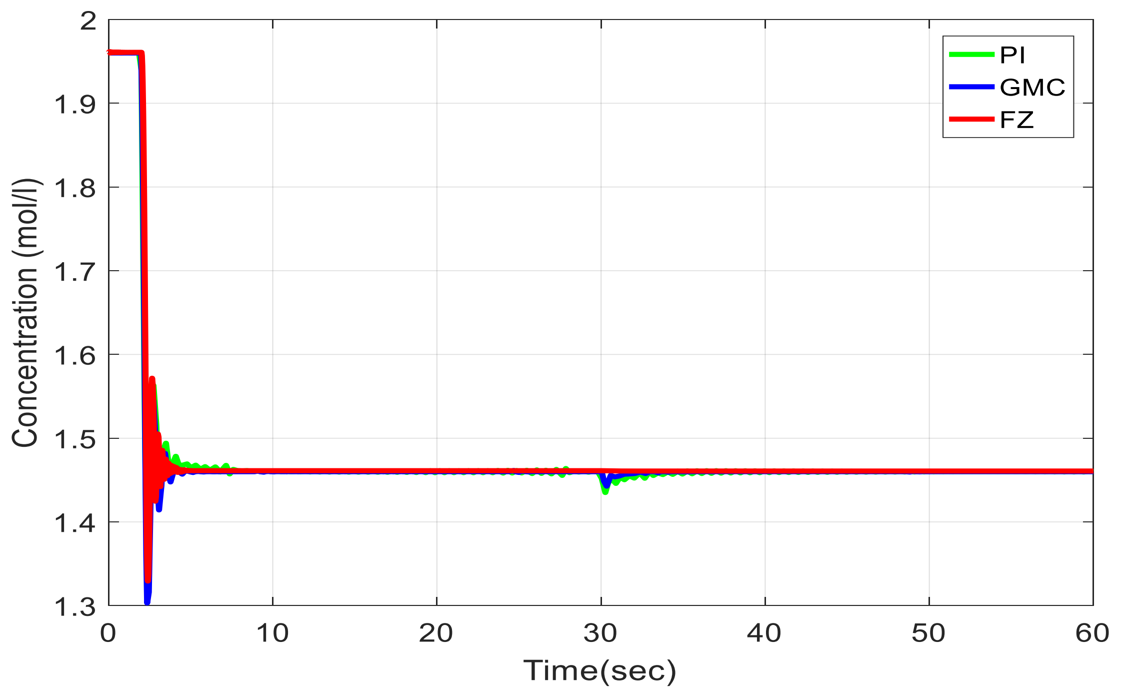


Figure 11. Concentration response of different controllers

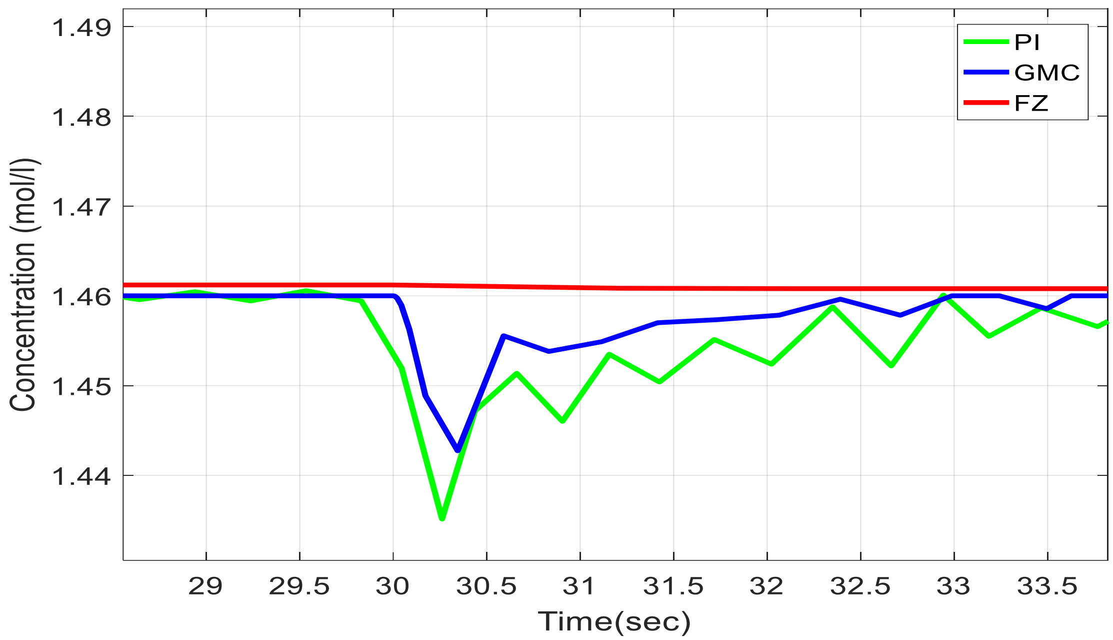


Figure 12. Enlargement of Concentration response of different controllers

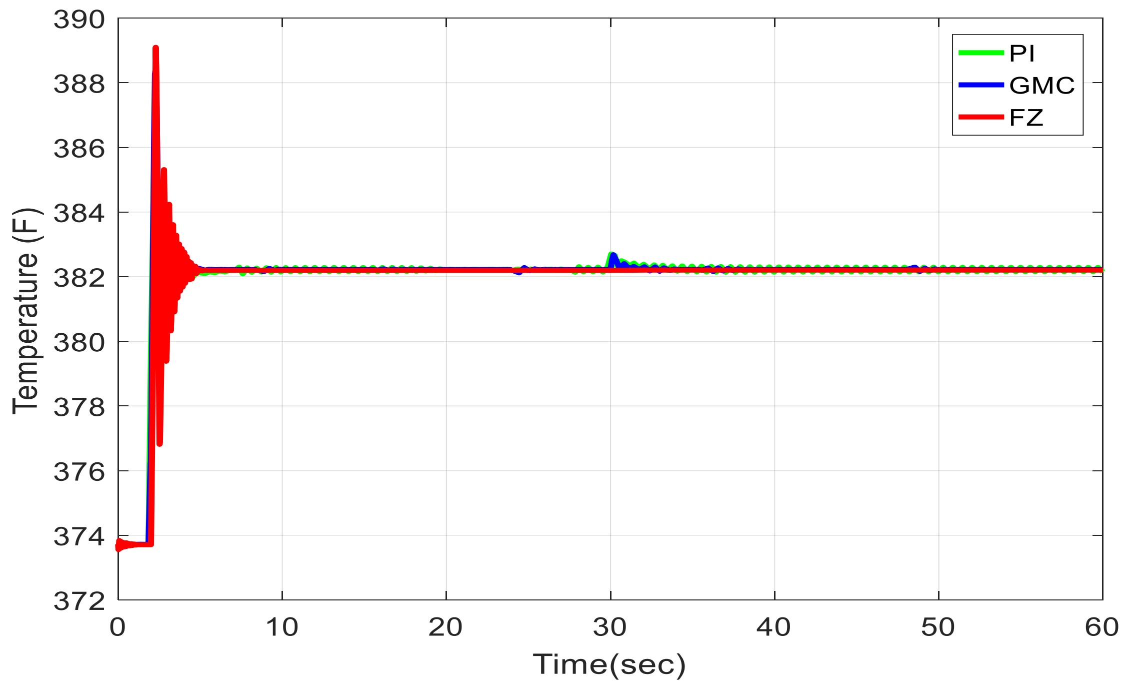


Figure 13. Temperature response of different Controllers

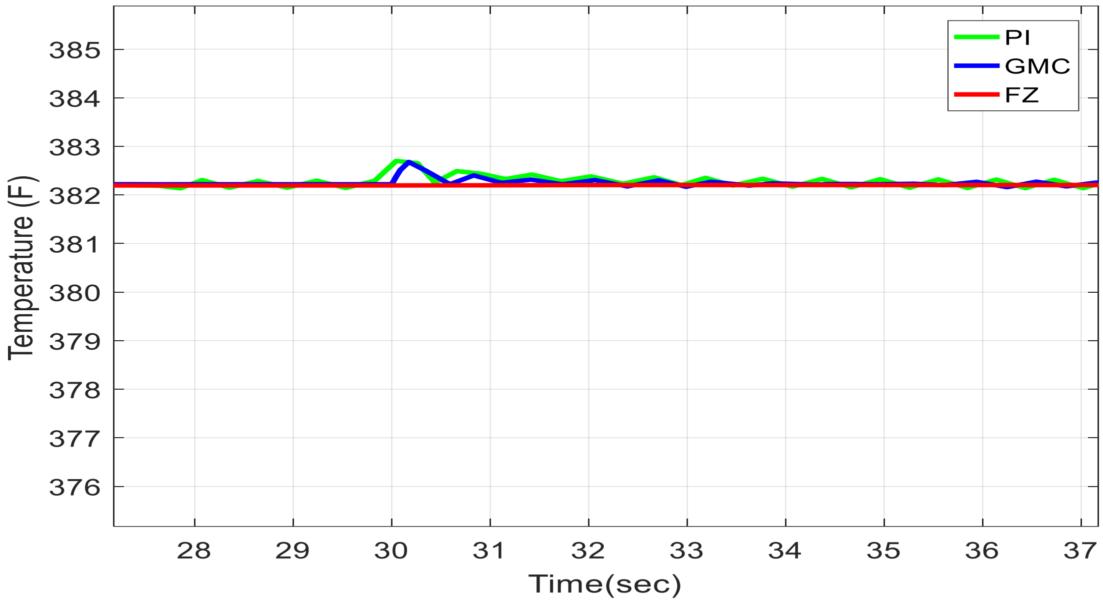


Figure 14. Enlargement of Temperature response of different Controllers

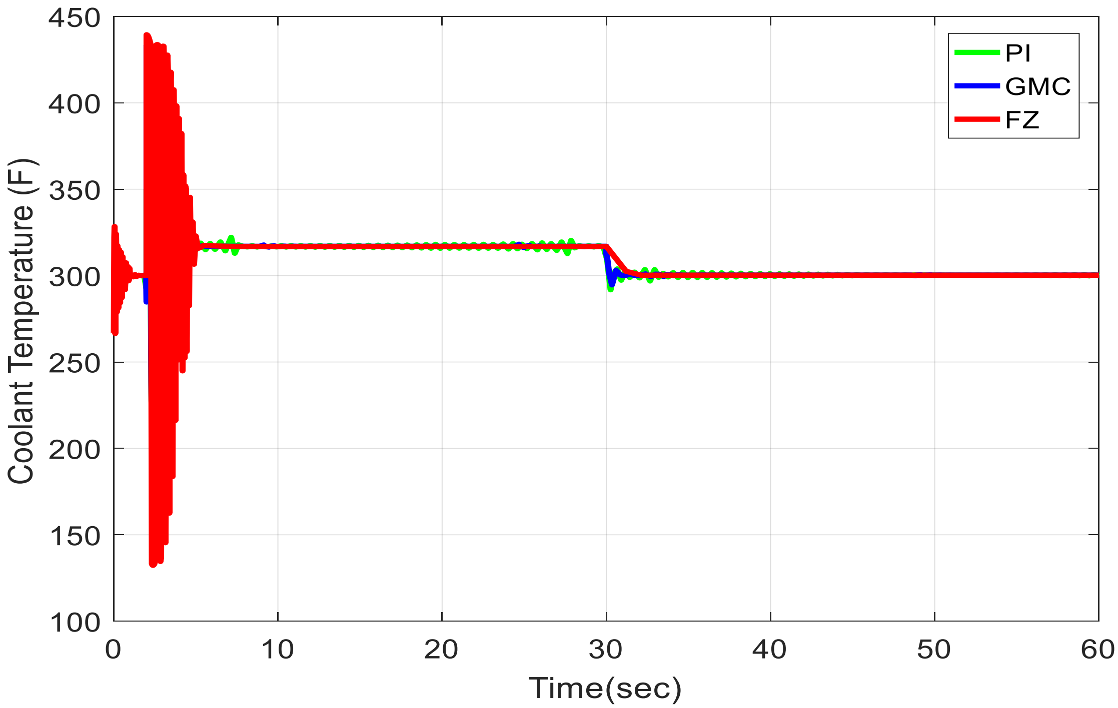


Figure 15. Coolant Temperature response for different Controllers

It is obvious that for both PI and GMC controllers an acceptable result can be achieved using conventional tuning methods, but it is very difficult to have a good membership function setting.

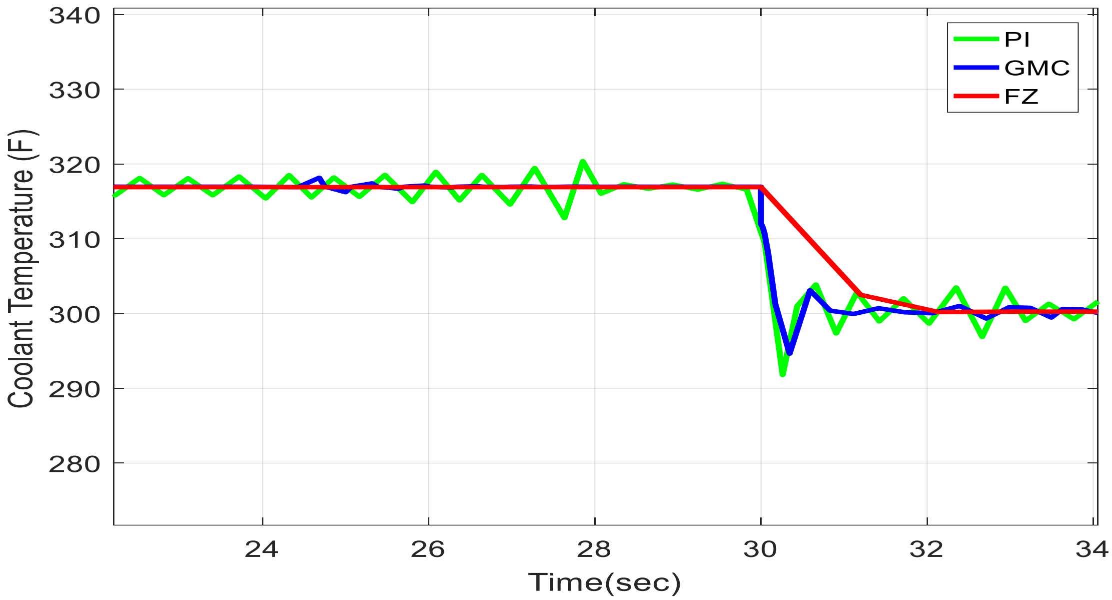


Figure 16. Enlargement of Coolant Temperature response for different Controllers

for fuzzy controller using trial and error. While, when applying simulated annealing the performance of the controllers in tracking the step change of the concentration from its initial value of 1.96 to 1.46 mol/l has been achieved. However, the controllers have the capability of eliminating the effect of the feed temperature disturbance from 300 F to 305 F on the concentration which is obvious at (30 sec) as can be seen in figures (11, 12). Moreover, it can be clearly seen in figures. (13, 15) that the Temperature and the coolant Temperature (Controller output) responses are changing according to their dependency to the concentration change, where, it is realized that at the initial concentration value, the temperature is 373.72 F, and the coolant Temperature is 300 F. When the concentration step change introduced at time (2 sec) where it has been reduced to 1.46 mol / l the temperature value rose to 382.22 F as well as the coolant temperature that rose to 316.9 F. However, at (30 sec) when the feed temperature disturbance was added, the controllers quickly overcame the disturbance and brought the temperature back to its steady state value, while the coolant temperature has dropped to 300.3 F which is the required controller value to keep the controlled parameter at its desired value. It is obvious that fuzzy controller response is a bit oscillatory at the start of the step change. Moreover, the fuzzy controller has better overcome of the feed temperature disturbance than the PI and the GMC controllers, but on the other hand they are much better in eliminating the steady state error. In order to investigate the performance of the controllers, and the capability of simulated annealing algorithm to find the best gains and membership function values that enhance the controller's performance. Figures below show the obtained results after these suggested enhancement additions.

The results obtained after increasing the controllers gain and membership functions by using a PI and GMC gain range of [-3000 3000] with a number of solutions of 1000, and increasing the fuzzy membership function for variable to 5 instead of 3, with a 5500 number of solutions for Fuzzy controller are depicted in figures [17, 18, 19, 20, 21]. The optimal IAE values respectively, are [0.162 0.164 0.139]. For PI, GMC, and fuzzy controllers, the optimum solutions were discovered at simulation times [670 534 2029], respectively.

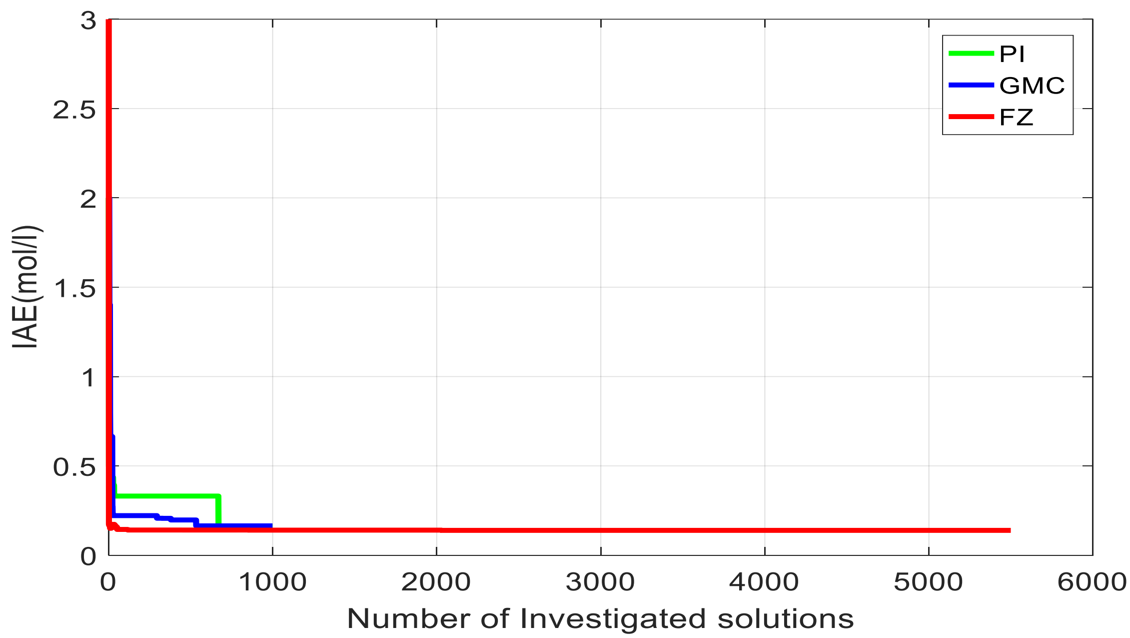


Figure 17. IAE obtained by SA for using different controllers

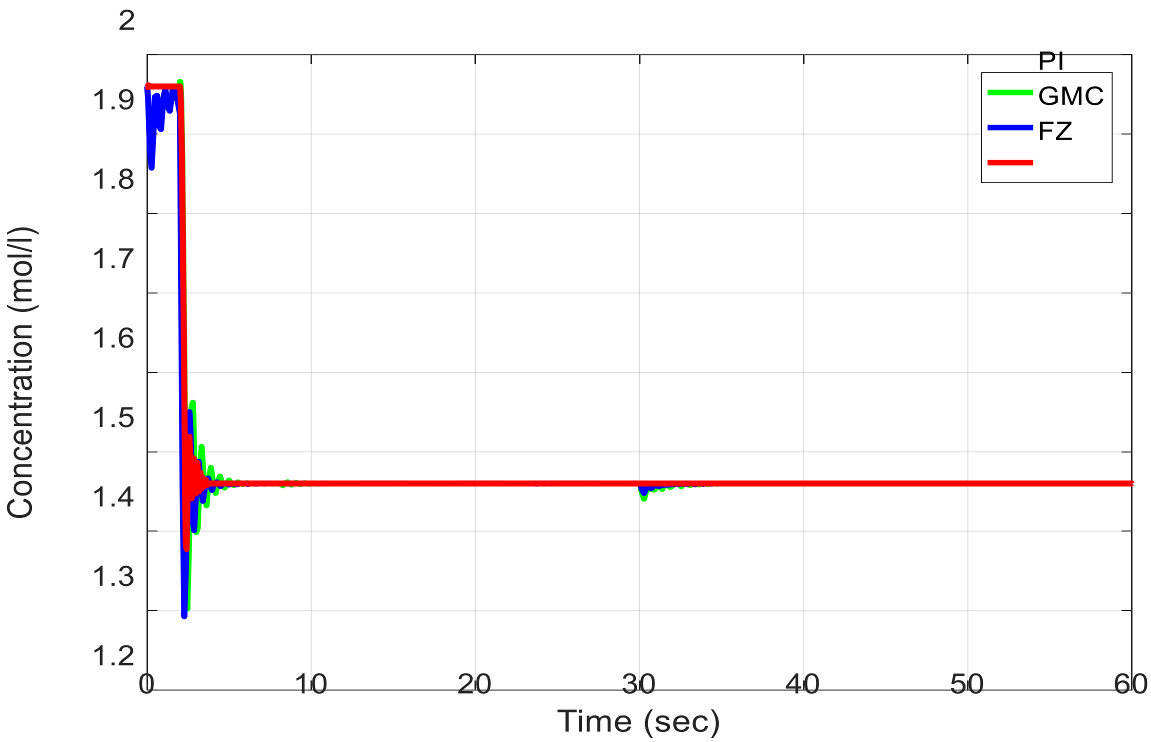


Figure 18. Concentration obtained by the three controllers

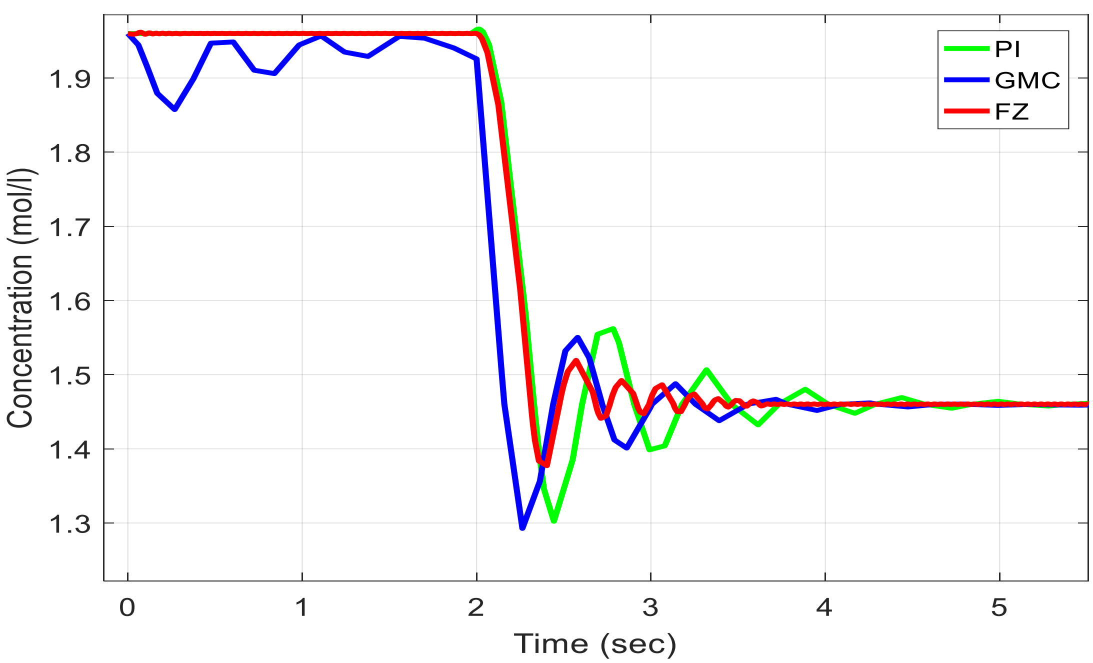


Figure 19. Enlargement of figure 18

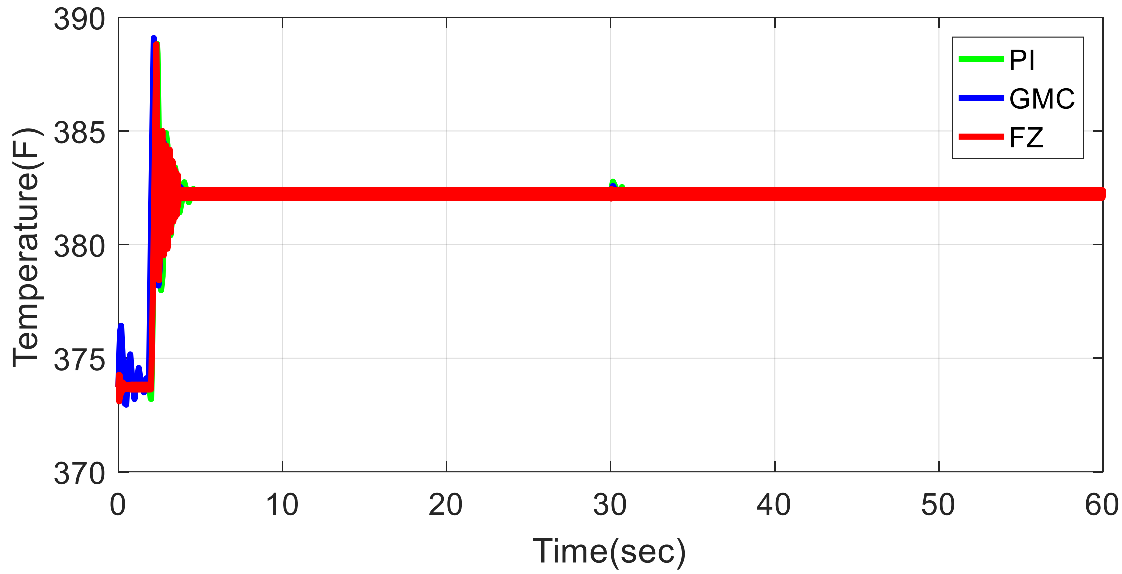


Figure 20, Temperature response for different controllers

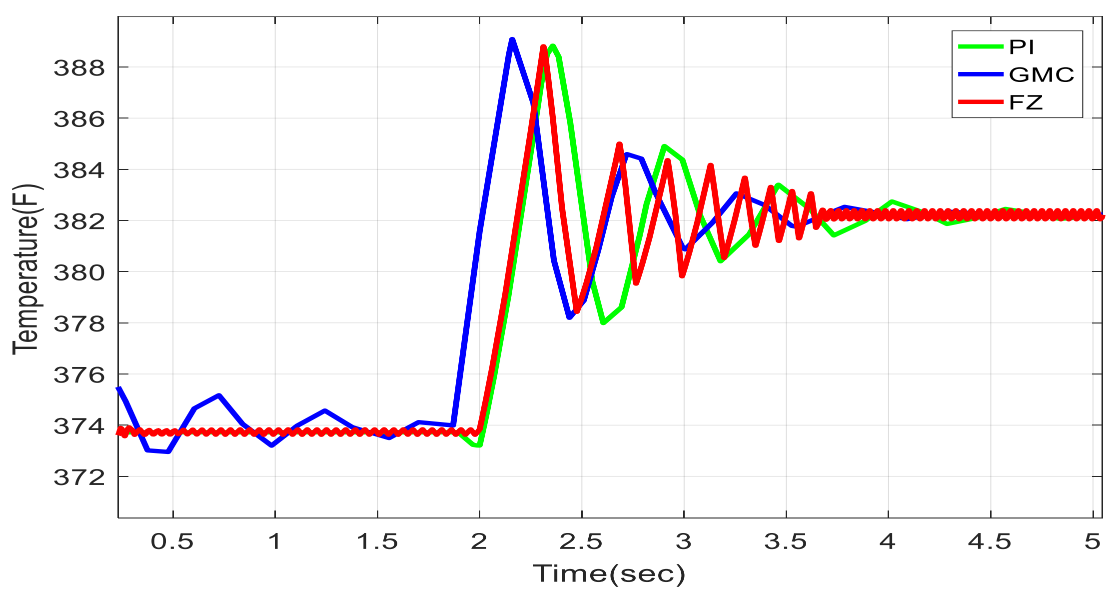


Figure 21. The enlargement of figure 20.

The results obtained after increasing the controllers gain and membership functions by using a PI and GMC gain range of [-3000 3000] with a number of solutions of 1000, and increasing the fuzzy membership function for variable to 5 instead of 3, with a 5500 number of solutions for Fuzzy controller. The optimal IAE values respectively, are [0.162 0.164 0.139]. For PI, GMC, and fuzzy controllers, the optimum solutions were discovered at simulation times [670 534 2029], respectively.

The following table shows the results obtained when tuning the controllers using conventional methods available in MATLAB optimization toolboxes and simulated annealing optimization technique illustrated above.

Table 1. Simulation results

Controller Type	IAE(mol/l) using conventional methods	IAE (mol/l) using simulated annealing (SA)	SA solutions number
PI	1.8992	0.1791 / 0.162	1000/ 1000
GMC	1.8606	0.1693/ 0.164	1000/1000
Fuzzy	4.2093	0.2048/ 0.139	3500/5500

6. Conclusion

Simulated annealing algorithm is a powerful stochastic optimisation technique, where it has proved it capability of finding the optimal controller parameters that can actively improve the performance of the controllers to its optimum. However, increasing the controllers gain or the number of membership function should incorporate with an increase in the number of simulations, to allow enough time for algorithm conversion, which is the character of simulated annealing algorithm of having a high conversion probability when the time increase with decrease of instantaneous temperature. Because of their inherited nonlinearity, fuzzy logic controllers are able to reject disturbances more effectively than PI and GMC controllers. However, the insurmountable steady state error is eliminated by PI and GMC controllers more effectively than the case of fuzzy controllers. Comparing the Integral Absolute Error which is the major criterion obtained using this algorithm to that obtained using conventional methods; it is evident that simulated annealing has found the best possible parameters that minimise the Integral Absolute Error to its minimum values, giving a better result of controller performance. The table above demonstrates the effectiveness of simulated annealing as a stochastic optimisation search method.

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