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Design, modeling and simulation of a square quartz resonant pressure sensor

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A B S T R A C T

This paper establishes the Finite Element Method FEM model of a practical Quartz beam resonator attached to a square diaphragm, which is used for measuring the pressure, based on sensing mechanism of a resonant Quartz pressure sensor. The relationship between the basic neutral frequency of the beam resonator and the measured pressure is calculated, analyzed and investigated by making use of the established FEM model. Some important qualitative and quantitative results on the natural frequency- pressure relationship of the beam resonator and the microsensor are obtained. Finally, based on the differential output scheme, a set of appropriate parameters of the sensing structure is determined, the frequency range is (661.839~892.208) kHz for the beam, which is located at the outer edge.

1 Introduction

During the last years, many specialists in sensor technology have noticed the rapid development in resonant sensors. Resonant sensor is based on the special sensing structure, which is operating at it resonating state. And the measured quantities can be detected by using the resonant frequency (or the natural frequency), phase shift or amplitude of the vibrating output signal for the sensing component [5].

The Quartz microstructure resonant sensors are noted for the advantages of a generalized resonant sensor, such as long term stability, high repeatability, low hysteresis and direct digital output. The mechanical properties of Quartz material are excellent, high strength, free from mechanical hysteresis, suitability to batch processing at low cost and the compatibility of mechanical and electrical properties. The dynamic characteristics of Quartz resonant sensors are much better than those of conventional sensors, due to their high working frequency [1]. In addition, the temperature characteristics of Quartz resonant sensors are much better than those of other important piezoresistive sensors. It is much easier to interface them with a microcontroller to develop intelligent sensors and the ability to batch process make low-cost, high performance sensors possible [1]. The design and construction of pressure sensors can be divided into on several different physical principles, which must be suitably selected to ensure that a sensor has the characteristics required. For instance, in certain cases a very high measurement accuracy will be needed, and in others, small dimensions and low weight; in the usual applications of weighing instruments, satisfactory zero stability is often necessary, and for pulse measurements it is all-important that the resonance frequency be high.

2 Sensor structure and operating principle

Figure 1 shows the structure of a Quartz resonant sensor for measuring pressure. The preliminary sensing unit is a square diaphragm. The measured pressure acts perpendicularly to the lower surface of the diaphragm and yields the stress. The final sensing unit is a beam, which is attached to the upper surface of the diaphragm. Moreover, the thickness of the beam h should be much less than the thickness of the diaphragm H, and the width of the beam should be less

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than the half length of the diaphragm A. Based on the above structural feature, an appropriate initial stress is applied along the axial direction of the beam, which is almost identical with the stress of the square diaphragm at the same position. Thus, the natural frequency of the beam is varied with the applied pressure which acts on the square diaphragm. Therefore, the pressure will be measured via the change in the natural frequency of the beam. In addition, the beam resonator has a very high Q factor because it can be packaged within a vacuum housing.

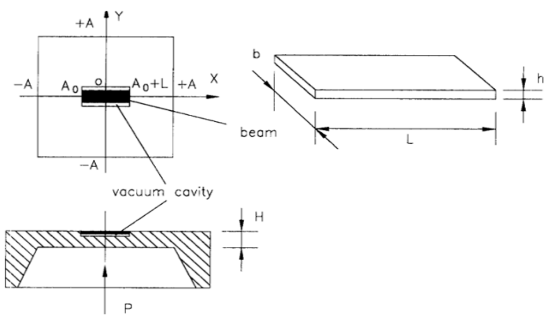

Fig. 1. Sensing structure of the pressure sensor.

3 Stresses on the upper plane of square diaphragm

According to the structural feature and the design demands for the pressure sensor, the square diaphragm is within the range of a small deflection. Then the differential equation can be written as follows [3]:

$$\frac{\partial^4 W(x,y)}{\partial x^4} + 2\frac{\partial^4 W(x,y)}{\partial x^2 \partial y^2} + \frac{\partial^4 W(x,y)}{\partial y^4} = \frac{P}{D_s} \quad (1)$$

$$D_s = \frac{EH^3}{12(1-\mu^2)}$$

Where W (x,y) Displacement of the square diaphragm under the applied pressure P, and Ds flexural rigidity of the square diaphragm E Young's modulus ,μ poisson ratio.

According to the built-in edge of the square diaphragm, its displacement can be assumed as follows:

$$W(x,y) = W_{\max} H\left(\frac{x^2}{A^2}-1\right)^2\left(\frac{y^2}{A^2}-1\right)^2 \quad (2)$$

Where W max ratio between the maximum normal displacement and the thickness of the square diaphragm, and A, H the Half-length and the thickness of the square diaphragm.

Substituting Eq. (2) into Eq. (1), the displacement W(x,y) can be obtained. Then stresses on the upper surface of the square diaphragm can be obtained.

$$\rho_x(x,y) = -\frac{49p}{96}\left(\frac{A}{H}\right)^2\left[\left(\frac{3x^2}{A^2}-1\right)\left(\frac{y^2}{A^2}-1\right)^2 + \mu\left(\frac{x^2}{A^2}-1\right)^2\left(\frac{3y^2}{A^2}-1\right)\right]$$

$$\rho_y(x,y) = -\frac{49p}{96}\left(\frac{A}{H}\right)^2\left[\left(\frac{3y^2}{A^2}-1\right)\left(\frac{y^2}{A^2}-1\right)^2 + \mu\left(\frac{y^2}{A^2}-1\right)^2\left(\frac{3x^2}{A^2}-1\right)\right] \quad (3)$$

Where σx(x,y),σy(x,y) Stresses of the square diaphragm.

4 Finite element model of the beam

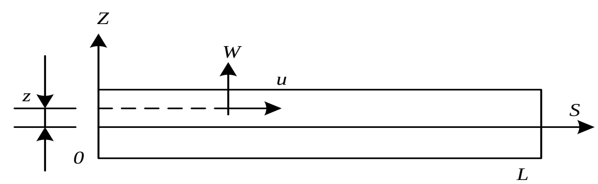

Fig. 5 Mathematical model of the beam

Figure 5 shows the mathematical model of the beam. The vibrating displacements of the beam at an arbitrary point are as follows:

$$\begin{cases} u(s,z,t) = -z\dfrac{dw(s)}{ds}\cos\omega t \\ w(s,t) = w(s)\cos\omega t \end{cases} \quad (4)$$

Where u (s,z,t)-w (s,t) Axial and normal vibrating displacements of the beam in Cartesian coordinate of the beam, and s,z Axial and normal coordinates of the beam in Cartesian coordinate of the beam.

Energy expressions of the beam resonator are as follows

The potential energy

$$U = \frac{Ebh^3\cos^2\omega t}{24}\int_S\left[\frac{d^2w(s)}{ds^2}\right]^2 ds \quad (5)$$

Where U Potential energy of the beam, and Sis the integrated length of the beam and b the width of the beam.

The kinetic energy

$$T = \frac{\rho bh\omega^2\sin^2\omega t}{2}\int_S[w(s)]^2 ds \quad (6)$$

Where T Kinetic energy of the beam, and ρ density of the sensing structure.

In addition, the initial potential energy of the beam, which is caused by $\sigma_s^0(s)$, is

$$U_0 = -\frac{bh\cos^2 \omega t}{2}\int_S \sigma_s^0(s)\left[\frac{dw(s)}{ds}\right]^2 ds \quad (7)$$

From Fig (1) and equation (3), according to the above analyses,

$$\sigma_s^0(s) = \sigma_x(x,0) = \frac{49P}{96}\left(\frac{A}{H}\right)^2\left[\left(\frac{3x^2}{A^2}-1\right)-\mu\left(\frac{x^2}{A^2}-1\right)^2\right] \quad (8)$$

Where σs0(s) Initial axial stress of the beam.

The following relation for the error cases:

$$\sigma_s^0(s) = \sin^2\beta(s)\sigma_x(x,y)+\cos^2\beta(s)\sigma_y(x,y) \quad (9)$$

Then the total potential energy of the beam is

$$U_T = U - U_0 \quad (10)$$

Where U0 Initial potential energy of the beam, which is caused byσs0(s), and UT Total potential energy of the beam.

In Eq. (7), if σ_s^0 (s) is a constant σ_s^0 the analytic relationship between the basic natural frequency and the initial axial stress can be directly obtained:

$$\omega = \frac{22.38h}{L^2}\sqrt{\frac{E}{12\sigma}}\sqrt{1+0.295\frac{\sigma_s^0 L^2}{Eh^2}} \quad (11)$$

Where ω in [rad/s], w (s) Natural frequency and its corresponding vibrating shape along the axial direction of the beam.

However, from Eqs. (3), (8) and (9), $\sigma_s^0(s)$ is varying. Therefore, the finite element equation of the beam resonator can be written as follows:

$$\left(\boldsymbol{K}-\omega^2\boldsymbol{M}\right)\boldsymbol{a}=0 \quad (12)$$

Where K-assembly stiffness Matrix, M-assembly Mass Matrix, the assembly nodal vector, consisting of all aj.

From Eqs. (11), (12), natural frequencies and the corresponding vibrating shapes of the beam resonator can be obtained.

5 Detection circuit system for pressure sensor

As the dimension of the resonant beam and the vibration amplitude is very small, the pickup signal is very weak and is submerged in the strong background noise [3]. To extract the weak vibrating signal, a detection circuit system is designed based on the lock-in amplification principle. Tests for the pressure sensor characteristics are realized by using this principle. Figure 2 is the block diagram of the circuit system.

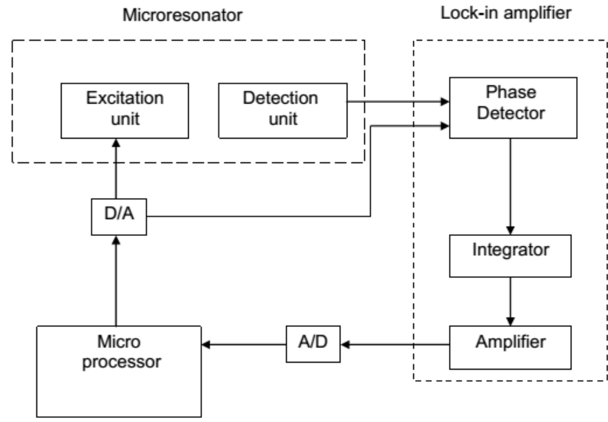

Fig .2. The block diagram of the detection circuit system

6 Calculation and discussion

In this paper, the main investigations are the varying laws of the pressure -frequency relationship for the beam resonator as the thickness H for the square diaphragm and the corresponding length of the beam L are varied. In order to get some generalized results of the Pressure -frequency relationship for the beam resonator, some related parameters are selected as follows:

The sensor is made of Quartz, $E = 7.2\times10^{10}$ Pa, $\rho = 2.2\times10^3$ kg/m3, μ =0.17. Moreover, the total element number of the beam N is 4 for FEM calculation.

The half-length and thickness of the square diaphragm are A=2.5 mm and H=0.14 mm, respectively. In addition, the width and thickness of the beam are b=40 µm and h=6µm.

7 Investigation of the frequency – pressure relationship

Define $f(P)$, $f(0)$ as the basic natural frequency of the beam for pressure P and for pressure $P=0$; $\Delta f = f(P)-f(0)$ as the variation of the basic natural frequency of the beam within $(0,P)$and $\beta = [f(P)-f(0)]/f(0)$ [%] as the relative variation or the sensitivity of the basic natural frequency for the beam within $(0,P)$ in

A- Location of beamon the upper plane of the square diaphragm

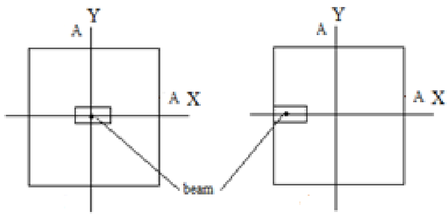

Fig. 3.Ideal locations of the beam at the diaphragm

Table I gives the beam's relationships between the frequency and the measured Pressure as the beam being located at different positions, and Table II gives the frequency variation and sensitivity of the beam corresponding to Table I.

From the Tables, some results can be obtained as follows:

As the beam is located at different positions of the upper plane of the square diaphragm along its radial direction, the sensitivity of the beam resonator is different. The best location for the beam resonator is at the outer edge for the square diaphragm, where the beam's sensitivity reaches the biggest, as shown in table II.

TABLE I. THE FREQUENCY OF BEAMAS *A*=4mm ,*L*=100 μm, *H*=0.15 mm

Pressure $\times10^5$ **(Pa)**	**Location of the beam**		
	(1.3,2.3)	**(2.5,3.5)**	**(4,5)**
0.0	661.839	661.839	661.839
0.1	656.432	663.693	697.839
0.2	629.549	665.265	722.839
0.3	620.152	667.413	747.839
0.4	609.602	669.682	771.708
0.5	599.773	671.830	792.708
0.6	588.994	673.174	812.708
0.7	576.384	675.492	832.708
0.8	567.840	677.519	852.708
0.9	558.957	679.384	872.208
1.0	547.417	681.283	892.208

TABLE II. THE VARIATION FREQUENCY AND SENSITIVITY OF BEAM

Position(mm)	**(1.3,2.3)**	**(2.5,3.5)**	**(4,5)**
Sensitivity	-17.2%	2.9 %	34.8 %
Variation(kHz)	-114.422	19.444	230.369

b-The thickness of the square diaphragm

The thickness of the beam h should be much less than the thickness of the diaphragm H, figure 4, Tables III and IV show the relationships between the frequency of beam and the pressure for different thicknesses of the diaphragm H.

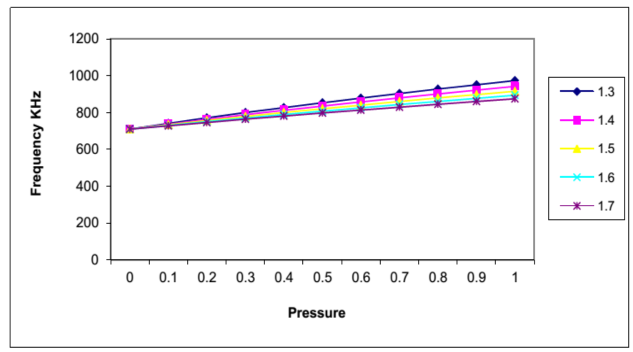

Fig 4. The relationships between the frequency of beam and the pressure

TABLE III. THE VARIATION FREQUENCY (KHZ) OF BEAM FOR THE DIFFERENT H and L

H $(\times10^{-4}$ m)	$(A\times10^{-3}$ m, $L\times10^{-3}$ m)		
	(5.0,1.12)	(5.0,1.1)	(5.0,1.08)
1.3	285.877	285.473	285.937
1.4	275.448	275.228	275.716
1.5	256.259	256.406	256.358
1.6	218.610	218.135	218.520
1.7	172.831	172.194	172.004

TABLE IV. THE SENSITIVITY OF BEAMFOR DIFFERENT H and L

H $(\times10^{-4}$ m)	$(A\times10^{-3}$ m, $L\times10^{-3}$ m)		
	(5.0,1.12)	(5.0,1.1)	(5.0,1.08)
1.3	42.4%	39.5%	36.7%
1.4	37.5%	34.9%	32.6%
1.5	33.4%	31%	28.8%
1.6	29.9%	27.8%	25.7%
1.7	27%	25%	23.1%

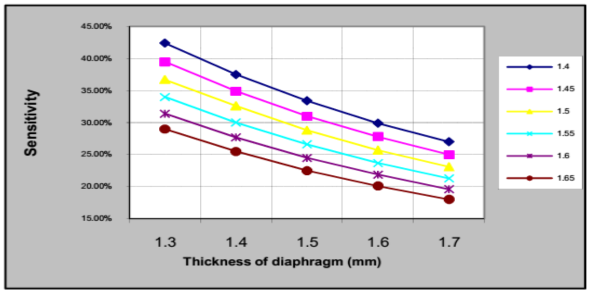

Fig 5. The relationships between sensitivity of Beam and the thickness of the diaphragm H

From the above Tables and Figures, the following results can be obtained:

The relative frequency variation (sensitivity) is increased as the thickness of the square diaphragm H decreasing for beam.

Then, the length of the beam L should be increased or the diaphragm thickness H should be decreased if the relative frequency variation (sensitivity) of the beam resonator is o be increased within a measurement range of pressure. Moreover, as the measurement range is narrow, the relative frequency variation (sensitivity) of the beam resonator should be high however as the measurement range is wider, the relative frequency variation (sensitivity) of the beam resonator should be narrower.

8 **Design and optimized parameters for pressure sensor**

Based on the differential output scheme of the Quartz resonant pressure sensor and some related criteria, a set of appropriate parameters for the above sensing structure of the sensor is determined for measuring the pressure, Table V gives the optimized parameters for the Quartz resonant pressures sensor.

TABLE V. THE OPTIMIZED PARAMETERS FOR THE PRESSURE SENSOR

The half length of square A	2.5 mm
Thickness H	0.14 mm
Length of beam L	100µm
Width of beam b	40 µm
Thickness oh beam h	6 µm
Frequency range for beam	(661.839~892.208) kHz

9 Conclusions

The Design, modeling and simulation for a resonant Quartz pressure sensor are carried out in this paper. The elementary sensing component of the sensor is the square diaphragm, and its final sensing component is the beam resonator which is attached to square diaphragm.

The main results obtained here are The finite element method model of the above Quartz complex sensing structure is established, based on its operating mechanism, Based on this model, the relationship between the basic natural frequency of the beam resonator and the applied pressure are calculated, analyzed and investigated, in detail, the sensitivity of the basic natural frequency to the measure pressure for the beam resonator will be increased as the thickness of the square diaphragm His decreased or the length of the beam is increased, The best selection for a beam when located at the outer edge of the square diaphragm, and the frequency range is (661.839~892.208) kHz for the beam.

References

G Stemme, Resonant Quartz sensors, Journal of Micromechanics and Microengineering, Volume 21, Number 2,2016.

M. Shaglouf shaeif.omer.Modeling and simulation on frequency characteristics of the silicon beam resonator attached to a square diaphragm for pressure sensor. Third International Conference on Technical Sciences (ICST2020), 28-30 November 2020, Tripoli – Libya.

Minhang Bao, Analysis and Design Principles of MEMS Devices, ELSEVIER, Amsterdam,2005

L. Cramphorn, J. loyd, N. F. Lepora, "Voronoi Features for Tactile Sensing: Direct Inference of Pressure Shear and Contact Locations", 2018 IEEE International Conference on Robotics and Automation (ICRA), pp. 2752-2757, 2018.

F. Cerini, M. Ferrari, V. Ferrari, A. Ardito, M. AzpeitaUrquia, R. Ardito, B. De Masi, M. Serzanti, P. DellEra, "MEMS force microactuator with displacement sensing for mechanobiology experiments", Proc. of AEIT International Annual Conference 7415224, pp. 1-6, 2015.

J. Rajagopalan, M.T.A. Saif, "MEMS Sensors and Microsystems for Cell Mechanobiology", J. Micromech. Microeng, vol. 21, no. 5, pp. 54002-54012, 2011.

S. Fan, G. Liu, M.Lee, Finite-element modeling and simulation on frequency characteristics of the Quartz beam resonator attached to an E-type round diaphragm for measuring the concentrated force, Sensors and Actuators, A63 pp.169~176 ,1997.